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SINGULARITY DETECTION USING HOLDER EXPONENT

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ABSTRACT

A signal processing technique called *Holder exponent* is presented to detect the presence of a discontinuity and when the discontinuity occurs in a dynamic signal. Wavelet transforms are incorporated with the Holder exponent to capture the time varying nature of discontinuities, and a classification procedure is developed to quantify when changes in the Holder exponent are significant. The proposed Holder exponent analysis is applied to acceleration response of a mechanical system with a rattling internal part. The experimental results demonstrate the effectiveness of the Holder exponent for identifying certain types of events that introduce discontinuities into the measured dynamic response data.

1 INTRODUCTION

The goal of this study is to develop a discontinuity detection technique based on the Holder exponent analysis, which can minimize unnecessary user interaction and can be potentially automated for the development of an autonomous continuous monitoring system. This application of the Holder exponent is not new in the analysis of time series data. For instance, Struzik 2001 uses the Holder exponent to characterize the underlying structure of a system that produces a time series of interest. The specific application is to financial data, where outliers and fluctuations in the Holder exponent value reveal interesting phenomena such as market crashes. Using the Holder exponent for discontinuity detection has also been shown to be useful in interpreting images (Shekarforoush et al. 1998). The edges in an image can be thought of as discontinuities and their identification can be used for finding abnormalities, removing noise, or even compressing the size of the image, because most of the information in an image is found in its edges. Holder exponents have even been used in one application of health monitoring. Hambaba and Huff 2000 use a wavelet transform to determine the Holder exponent value of a gear response at different scale levels. By fitting an Auto Regressive Moving Average (ARMA) model to the wavelet-transformed data, analysis of the residual error is used to indicate the presence of fatigue cracks in the gear. Peng et al. 2002 examine shaft orbits using the wavelet modulus maxima. The wavelet modulus is the absolute value of the wavelet transform and its maxima are ridges of high-valued coefficients that progress through the time-frequency plane. The Holder exponent values are extracted only for these maxima lines and then their distribution is used as input features to a neural network, which classifies the shaft orbit (including fault classification). These two applications are very different from the one presented in this paper. This paper will use the wavelet transform to obtain a time-based local Holder exponent function. Fluctuations in this function as demonstrated in Struzik 2001 will be useful for understanding and identifying outliers in the data. Hambaba and Huff 2002, on the other hand, are looking at the global regularity of the data at various scales and Peng et al 2002 use the Holder exponent at specific points in time as a feature, rather than its variation in time.

2 HOLDER EXPONENT ANALYSIS

A Holder or Lipschitz exponent, which provides a measure of a signal's regularity, is presented to detect the presence of a discontinuity and when the discontinuity occurs in a dynamic signal. The regularity of a signal is defined as the number of continuous derivatives that the signal possesses. First, the time varying nature of the Holder exponent is obtained based on a wavelet transform. Because discontinuity points have no continuous derivatives, these points are identified by locating time points where the Holder exponent value suddenly drops. Next, an automated classifier is developed to quantify when changes in this Holder exponent are significant.

Wavelets are mathematical functions that decompose a signal into its constituent parts using a set of wavelet basis functions. This decomposition is very similar to Fourier transforms, which use dilations of sinusoids as the bases. The family of basis functions used for wavelet analysis is created by both *dilations* (scaling) and *translations* (in time) of a "mother wavelet", thereby providing both time and frequency information about the signal being analyzed. There are many different functions that can be called wavelets. In this study, the Morlet wavelet is used for the family of basis functions. The resulting coefficients from the wavelet transform of a time domain signal, such as the acceleration response of a structure, can be represented in a two-dimensional time-scale map. Examination of the modulus of the wavelet transform shows that many of these coefficients are very small in magnitude. Large magnitude components, termed modulus maxima, will be present at time points where the most change in the signal has occurred. Jumps or singularities in the signal can therefore be identified by the presence of modulus maxima at specific time points in the wavelet map. Singularities are distinguishable from noise by the presence of modulus maxima at all of the scale levels for a given time point. Noise will produce maxima at the finer scales, but will not persist to the coarser scales.

Mallat and Hwang 1992 first introduced a method for detecting singularities in a signal by examining the evolution of the maxima of the modulus of the wavelet transform across the scales. The decay of this maxima line can then be used to determine the regularity of the signal at a given time point. A less time consuming alternative to the extraction of the maxima line is to simply look at the decay of the wavelet modulus across the scales at a given time point. Points of large change in the signal will have large coefficients at all the different scales, thus having little decay. The measure of this decay is the Holder exponent of the signal at a given time point.

The Holder regularity is defined as follows. Assume that a signal $f(t)$ can be approximated locally at t_0 by a polynomial of the form (Struzik 2001):

$$\begin{aligned} f(t) &= c_0 + c_1(t-t_0) + \ldots + c_n(t-t_0)^n + C\left|t-t_0\right|^\alpha \\ &= P_n(t-t_0) + C\left|t-t_0\right|^\alpha \end{aligned} \tag{1}$$

where P_n is a polynomial of order n and C is a coefficient. The term associated with the exponent α can be thought of as the residual that remains after fitting a polynomial of order n to the signal, or as the part of the signal that does not fit into an $n+1$ term approximation. The local regularity of a function at t_0 can then characterized by this "Holder" exponent:

$$\left|f(t) - P_n(t-t_0)\right| \le C\left|t-t_0\right|^\alpha \tag{2}$$

In order to detect discontinuities, a transform is needed that ignores the polynomial part of the signal. A wavelet transform that has n-vanishing moments is able to ignore polynomials up to order n. Transformation of Equation (2) using a wavelet with at least n vanishing moments then provides a method for extracting the values of the Holder exponent in time:

$$\left| Wf(u,s) \right| \leq Cs^{\alpha} \tag{3}$$

where $Wf(u,s)$ is the wavelet transform at time translation u and scale s. The wavelet transform of the polynomial is zero and so what remains is a relationship between the wavelet transform of $f(t)$ and the error between the polynomial and $f(t)$, which relates to the regularity of the function. When a complex wavelet such as the Morlet wavelet is used, the resulting coefficients are also complex. Therefore, the magnitude of the modulus of the wavelet transform, called the scalogram, must be used to find the Holder exponent.

The steps for calculating the Holder exponent in time are as follows. First, take the wavelet transform of the given signal and take the absolute value of the resulting coefficients to obtain the wavelet transform modulus:

$$\left| Wf(u,s) \right| = \left| \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{s}} \psi^{*}\left(\frac{t-u}{s} \right) dt \right| \tag{4}$$

Arrange the coefficients in a two-dimensional time-scale matrix. One dimension of the time-scale matrix (u) represents a different time point in the signal, and the other dimension denotes a different frequency scale (s). Take the first column, which represents the frequency spectrum of the signal at the first time point, and plot the log of it versus the scales, s, at which the wavelet transform was calculated. This procedure can be shown mathematically by taking the log of each side of Equation (3):

$$\log \left| Wf(u,s) \right| = \log(C) + \alpha \log(s) \quad \text{and} \quad m = \frac{\log \left| Wf(u,s) \right|}{\log(s)} = \alpha \tag{5}$$

Ignoring the offset due to the coefficient C, the slope m is then the decay of the wavelet modulus across its scales. Negating the slope will give the decay versus the frequencies of the transform rather than the scales, due to the inverse relationship between scale and frequency. The Holder exponent α is then simply the slope m. This is the Holder exponent for the first time point in the signal. To find the Holder exponent at all time points, repeat this process for each time point of the wavelet modulus matrix.

Once the exponent α is calculated as a function of time, the measurement of the regularity can be used to detect discontinuities in a signal. The easiest way to identify a discontinuity is by looking for a distinct downward jump in the regularity versus time plot. A discontinuous point should have a Holder exponent of zero, but resolution limitations of the wavelet transform will result in slightly different values. So, identifying areas where your Holder exponent dips from positive values towards zero, or below, will identify when the discontinuities in the signal occur. In this paper, an automated classifier is created to detect the presence of discontinuities in the signals by identifying drops in the Holder exponent in time. The previous investigation by the authors indicates that looking at the depth of a drop in the Holder exponent is an effective way of assessing a discontinuity (Robertson et al. 2002).

A threshold is set such that any drops that exceed this threshold are labeled as discontinuities. The threshold value is set using a portion of the data known to contain no discontinuities, and this portion of data is termed "normal" data. The procedure starts by finding all the local maxima and minima of the Holder exponents in time for the normal signal. Then, drops in the Holder exponent values are calculated as the difference between a given minimum and the maximum immediately preceding it. Next, the threshold is determined by finding the largest drop under "normal" conditions and amplifying this number by a factor of 1.5. The procedure for determining the depths of the drops in the Holder exponent function is then repeated on the remaining data of interest. If any of the drops are 50% deeper than the biggest drop in the normal data, the time point is identified as a discontinuity location. It should be noted that this amplification factor is application specific, and this threshold can be altered to be more or less restrictive, based on the needs of the application. For the example presented in this paper, the value of 1.5, however, serves our purpose well. In some instances, the dips in the Holder value are jagged with small oscillations. This property makes the estimation of the dip's depth a difficult task because the algorithm relies on comparing the local maxima and minima, which will now also appear in the small fluctuations in the dip itself. Therefore, it was decided to add the option of smoothing the Holder exponent values before performing the discontinuity detection algorithm by applying a low-pass moving average (MA) filter.

3 APPLICATION

The effectiveness of the Holder exponent for detection discontinuities in signals is demonstrated using the acceleration response of a mechanical structure subjected to a harmonic base excitation. Applications of the proposed Holder exponent analysis to other structures can be found in Robertson et al 2002 and Sohn et al 2002. The defense nature of the test structure precludes a detailed description of its geometry or material properties. Instead a structure that is conceptually similar is schematically shown in Figure 1. The exterior container of the system is horizontally excited at 18 Hz, and the nonsymmetrical bumpers attached to two interior side walls of the container cause the internal mass to exhibit a rattle during one portion of the harmonic excitation.

Figure 2 shows the response of the structure at three different excitation levels measured by accelerometers mounted on the outer structure in the in-axis and off-axis directions. The placement and orientations of the in-axis and off-axis accelerometers are shown in Figure 1. The rattle produced by these impacts is evident in the sensor measurements that are off-axis from the excitation. The short oscillations of increased magnitude in these measurements are indicative of the rattle. For the lowest excitation level, the rattling is occurring near 5.33 and 10.89 milliseconds. Similar rattling can be observed for the intermediate and the highest excitation levels near 1.34, 6.88, and 12.39 milliseconds, and 0.25, 5.80, and 11.33 milliseconds, respectively. These same oscillations are not readily apparent in the in-axis data, particularly if one does not have the off-axis measurements for reference. The examination of the data from all three input levels shows that one can only see the rattle clearly in the off-axis measurements, and the lowest excitation level had the lowest signal to noise ratio, thus making observation of the rattle more difficult. The purpose of this application is to determine whether the presented technique can be used to identify the time when the rattle is occurring by looking only at the in-axis response measurements. For the acceleration signals analyzed in this example, 4096 acceleration time points are recorded for 0.125 seconds resulting in a sampling frequency of 3276 Hz.

First, a wavelet transform (scalogram) is applied to the in-axis response of all three levels as shown in Figure 3. A complex-valued Morlet wavelet with length 16 is used for the scalogram, and 256 scales are used between 0 Hz to the Nyquist frequency (1638 Hz). A larger width wavelet function increases the overall frequency resolution of the transform while decreasing the time resolution. The coarse time resolution acts as a smoothing filter decreasing the amount of spurious oscillation in the Holder exponent and bringing out changes associated with true discontinuities. To minimize the end effect of the wavelet transform, a continuous wavelet transform with mirroring is used in this example. The program for this mirrored wavelet computation is available at www.irccyn.ec-nantes.fr/FracLab/Fraclab.html as part of Fractal Analysis Software, copyrighted by INRIA. The time locations of the rattling phenomena are clearly with the naked eye in Figure 3.

The next step is to transfer this visual interpretation of the images to a more automated identification procedure. For this purpose, the Holder exponent was extracted from the scalogram. Figure 4 shows the Holder exponent obtained from the previous scalogram in Figure 3. To smooth the plot of the Holder exponent, averaging with a moving window size of 8 was applied. The singularities associated with the rattle are clearly visible in this plot at each time they occur during the oscillatory cycles. Though the dips in the Holder exponent shown in Figure 4 are fairly apparent to the naked eye, identification of them using an automated procedure is more difficult. The discontinuity classification algorithm described in the previous section was used to identify the locations of the rattle. The first 1000 time points of the lowest level response was used as the "normal data" to establish the threshold value for discontinuity detection. The threshold value was then set at 150% of the largest dip in the normal data. Filtering using the moving average method effectively removed the spurious oscillations in the dips and allowed for a successful detection of the discontinuities as shown by the circles in Figure 4.

4 CONCULSIONS

In this study, a Holder exponent analysis is successfully applied to the acceleration response of a mechanical system subjected to a harmonic excitation with a rattling internal part. Furthermore, a discontinuity classifier is developed to automate the identification procedure of discontinuities. The simplicity and data driven nature of the proposed approach makes it very attractive for embedding the discontinuity algorithm into a digital signal processing chip or field programmable gate array, which can be an integrated part of an intelligent sensor unit with micro-electromechanical system (MEMS) sensors, a wireless telemetry, on-board computation power and a battery.

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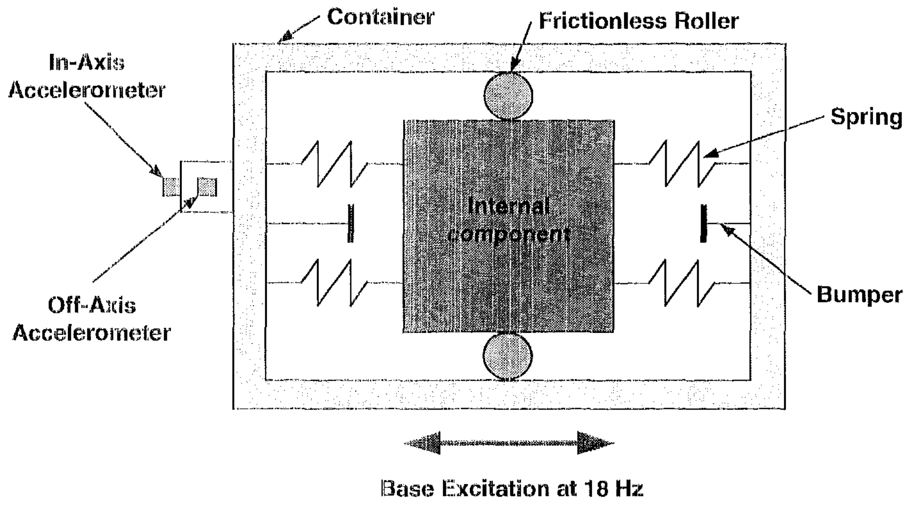

Figure 1: A schematic diagram of a mechanical test structure that has a loose internal part and non-symmetric bumpers

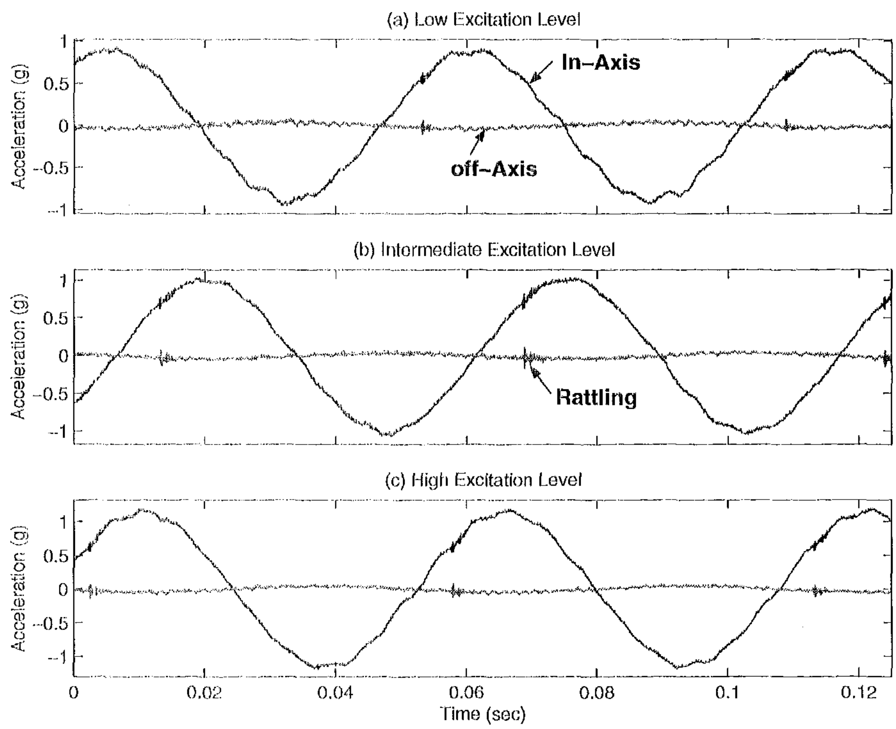

Figure 2: Acceleration response of the test structure at three base excitation levels as measured in the in-axis and off-axis directions by accelerometers mounted on the outer structure

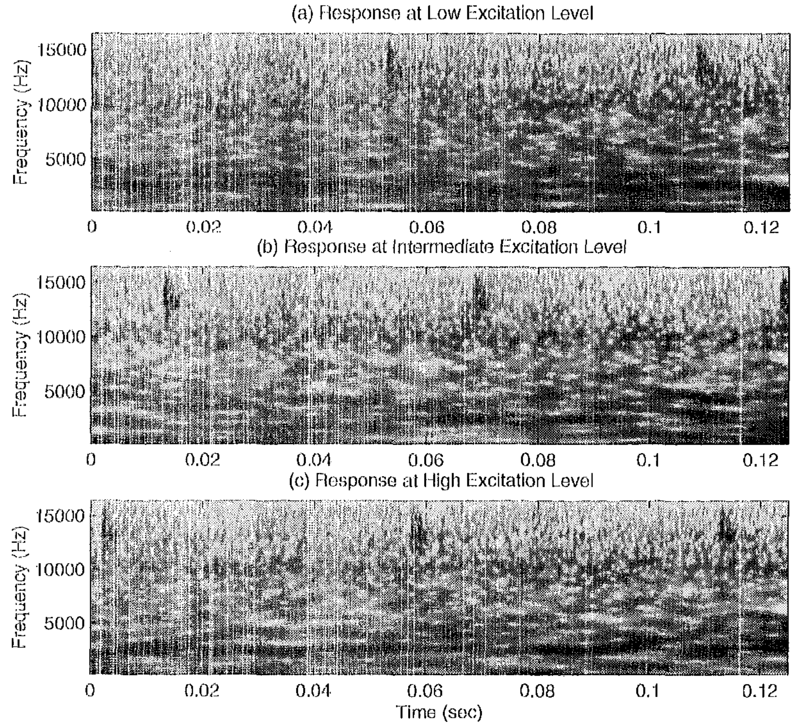

Figure 3: The scalogram for the in-axis acceleration data subject to the three-excitation levels

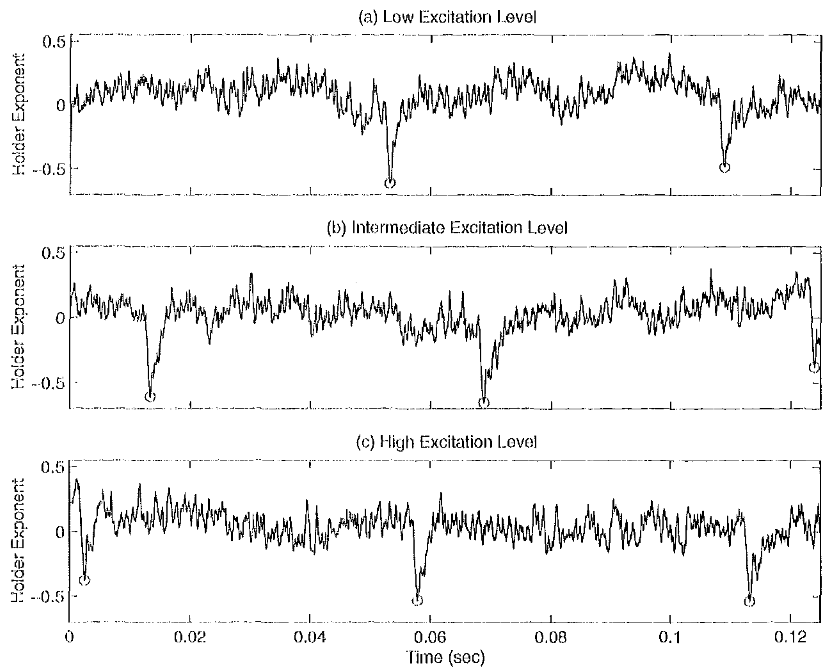

Figure 4: The Holder exponent extracted from the scalogram of the in-axis acceleration data subjected to the three-excitation levels